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Quantum Cosmology in CGBD Theory

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We apply the theory developed in quantum cosmology to a model of charged generalized Brans–Dicke gravity. This is a quantum model of gravitation interacting with a charged Brans–Dicke type scalar field which is considered in the Pauli frame. The Wheeler–DeWitt equation describing the evolution of the quantum Universe is solved in the semiclassical approximation by applying the WKB approximation. The wave function of the Universe is also obtained by applying both the Vilenkin-like and the Hartle–Hawking-like boundary conditions. We then make predictions from the wave functions and infer that the Vilenkin's boundary condition is more reasonable in the Brans–Dicke gravity models leading a large vacuum energy density at the beginning of the inflation.

KEY WORDS: quantum cosmology; Brans–Dicke gravity.

1. HAMILTONIAN FORMULATION OF CGBD GRAVITY

To start with, consider the original version of the Einstein–Hilbert action for the Brans–Dicke gravity with a real scalar field

$$S = \int d^4x \sqrt{-\gamma} \left(\Phi \tilde{R} + \frac{\omega \gamma^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi}{\Phi} \right) \tag{1.1}$$

where $\gamma = \det \gamma_{\mu\nu}$, $\gamma_{\mu\nu}$ is the Jordan metric, Φ is the Brans–Dicke scalar field, and ω is the dimensionless Brans–Dicke coupling constant. Here we make a transformation for the scalar field,

$$\Phi = \frac{\varepsilon \varphi^2}{2} \tag{1.2}$$

$$\omega = \frac{1}{4\varepsilon} \tag{1.3}$$

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Then the Brans–Dicke action (1.1) becomes a form which is more easily handled afterward

$$S = \int d^4x \sqrt{-\gamma} \left(\frac{\varepsilon\varphi^2}{2} \tilde{R} + \frac{1}{2}\gamma^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \right) \tag{1.4}$$

In Cho's paper (Cho, 1992), the spin-2 massless graviton is represented only by the Pauli metric. By means of a conformal transformation, $g_{\mu\nu} = e^{\alpha\sigma}\gamma_{\mu\nu}$, where σ is the dilaton scalar field, the fundamental assumption that the gravitational interactions are generated by the massless spin-2 gravitons and are then realized in Brans–Dicke gravity.

On the other hand, the Jordan frame formulation of a scalar–tensor theory is not viable because the energy density of the gravitational scalar field present in the theory is not bounded from below (violation of the weak energy condition (Wald, 1984)). The system therefore is unstable and decays toward a lower and lower energy state ad infinitum (Faraoni *et al.*, 1998; Magnano and Sokolowski, 1994). However, the Pauli metric formulation of scalar–tensor theories is free of the problem.

The example illustrated in Faraoni and Gunzig (1999) shows, in a staightforward way, the violation of the weak energy condition by wave-like gravitational field in Brans–Dicke theory formulated in the Jordan frame, and the viability of the Pauli frame counterpart of the same theory. The example is not academic, since an infrared catastrophe for scalar gravitational waves would have many observational consequences. One example studied in the astronomical literature consists of the amplification effect induced by scalar–tensor gravitational waves on a light beam, which differs in the Jordan and in the Pauli frame (Bracco and Teyssandier, 1998; Faraoni, 1996; Faraoni and Gunzig, 1998). Within the classical context, scalar–tensor theories must be formulated in the Pauli frame, not in the Jordan one.

In most of the current literature, the matter field is constrained to be real scalar field with just one degree of freedom. Actually, the standard quantum cosmology as well as the classical inflationary cosmology is based on the Friedmann cosmology in the presence of this real scalar fields. However, it is evident that a complex scalar field has more physical sense because it corresponds to a matter hydrodynamical field (Khalatnikov, 1992). In a series of papers, Khalatnikov and Mezhlumian (Khalatnikov and Schiller, 1993) showed how one can handle the new degree of freedom introduced by the phase of a complex scalar field. We are now about to investigate the effect on the wave function and its prediction when a complex scalar field is coupled to the gravity and the physical metric is chosen to be the Pauli metric as mentioned before.

Consider a model of a Brans–Dicke scalar field with a global $U(1)$ charge coupled to gravity and described by the following action:

$$S = \int d^4x \sqrt{-\gamma}(\varepsilon\varphi^*\varphi\tilde{R} + \gamma^{\mu\nu}\partial_\mu\varphi^*\partial_\nu\varphi) \tag{1.5}$$

where φ^* is the conjugate field of φ. The Hilbert–Einstein action (1.5) represents an extension of the "old" Brans–Dicke action (1.1) with a complex scalar field of the form

$$\varphi = \frac{\tilde{\phi}}{\sqrt{2}} e^{i\tilde{\theta}} \tag{1.6}$$

where $\tilde{\phi}$ is the absolute value of the complex scalar field and $\tilde{\theta}$ is its phase. With this substitution, the action for the charged Brans–Dicke scalar field (1.5) becomes

$$S = \int d^4x \sqrt{-\gamma} \left(\frac{1}{2} \varepsilon \tilde{\phi}^2 \tilde{R} + \frac{1}{2} \gamma^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} + \frac{1}{2} \tilde{\phi}^2 \gamma^{\mu\nu} \partial_\mu \tilde{\theta} \partial_\nu \tilde{\theta} \right) \tag{1.7}$$

Up to now, the actions (1.5) and (1.7) are formulated in the Jordan metric. To obtain the Einstein–Hilbert action of the charged Brans–Dicke theory in the physical Pauli metric, we introduce a real Brans–Dicke dilaton field σ according to the transformation proposed in Cho's paper (Cho, 1992).

$$\tilde{\theta} = e^{\alpha\sigma/2} \tag{1.8}$$

where α is the normalization constant which has a value

$$\alpha = \sqrt{\frac{2}{2\omega + 3}} \tag{1.9}$$

and the Weyl rescaling metric

$$g_{\mu\nu} = \frac{e^{\alpha\sigma}}{4\omega} \gamma_{\mu\nu} \tag{1.10}$$

where $g_{\mu\nu}$ is now called the Pauli metric. Hence, from (1.7) and the generalization to include the potential term of the scalar field, we obtain the final version of the action for the charged generalized Brans–Dicke gravity (CGBD gravity)

$$S = \int d^4x \sqrt{-g} \left(R + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \frac{1}{2} g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta + V(\sigma) \right) \tag{1.11}$$

where the potential of the dilaton field takes a form of $V(\sigma) = \Lambda$, where Λ is a constant energy density, the action reduces to the theory of a charged Brans–Dicke field in a background with cosmological constant Λ. Notice that the gravitational coupling to the dilatonic matter becomes normal and it is now the dilatonic matter which moves along the geodesic determined by the Pauli metric (Cho, 1992). In other words, the nonminimal coupling in Jordon metric case becomes now the minimal coupling of the scalar field in Pauli metric case.

We shall consider the minisuperspace model of the spatially homogenous and isotropic Universe. The metric of the space-time is suggested to be described by the Friedmann–Robertson–Walker line element,

$$ds^2 = -N^2\,dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right) \tag{1.12}$$

where as usual notation $a(t)$ is the scale factor of the Universe and $k = -1, 0, 1$ represents the curvature of the spatial section. In our model, the Universe is assumed to be closed and k is chosen to be $+1$. Substituting the 4-metric into the total action (1.11), we obtain

$$S = 2\pi^2 \int dt\, N\left(\frac{-6a\dot{a}^2}{N^2} + 6a + \frac{a^3}{2N^2}\dot{\sigma}^2 + \frac{a^3}{2N^2}\dot{\theta}^2 - a^3 V(\sigma)\right) \tag{1.13}$$

Obviously, the corresponding conjugate momenta are

$$\pi_a = \frac{\partial L}{\partial \dot{a}} = \frac{-12a\dot{a}}{N} \tag{1.14}$$

$$\pi_\sigma = \frac{\partial L}{\partial \dot{\sigma}} = \frac{a^3\dot{\sigma}}{N} \tag{1.15}$$

$$\pi_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{a^3\dot{\theta}}{N} \tag{1.16}$$

Luca (1994) considered that the phase variable θ is a cyclical one and its conjugate momentum π_θ is a conserved quantity. It is the classical charge of the Universe which plays the role of the new quasi-fundamental constant.

$$Q = \frac{a^3\dot{\theta}}{N} \tag{1.17}$$

Consider the canonical formalism and by using the relation (1.17), rewrite the action (1.13) in the following form:

$$S = 2\pi^2 \int dt\,(\pi_a\dot{a} + \pi_\sigma\dot{\sigma} - NH) \tag{1.18}$$

The variation of the action with respect to the lapse function N giving the Hamiltonian constraint

$$\frac{\partial S}{\partial N} = 0 \tag{1.19}$$

$$H = 0 \tag{1.20}$$

where the Hamiltonian is explicitly written as

$$H = \left(\frac{-\pi_a^2}{24} + \frac{\pi_\sigma^2}{2a^2} - U(a,\sigma)\right) = 0 \tag{1.21}$$

which is the well-known Wheeler–DeWitt equation defined on the minisuperspace with only a and σ as the variables because the third variable θ is collected in the conservation of the momentum conjugate. The function $U(a, \sigma)$ is the potential term and takes a form

$$U(a, \sigma) = 6a^2 - \frac{Q^2}{2a^2} - a^2 V(\sigma) \tag{1.22}$$

Before going to have quantization of Hamiltonian constraint and work out the wave function of the Universe, it is better and illustrative to firstly investigate the geometry of the Euclidean regions because of the presence of the charge Q. From (1.21) we can see that the Euclidean region occurs when the following condition takes place:

$$U(a, \sigma) > 0 \tag{1.23}$$

This is the classically forbidden region in which the classical trajectories cannot enter. The boundary of the Euclidean region is given by

$$U(a, \sigma) = 0 \tag{1.24}$$

2. GEOMETRY OF EUCLIDEAN REGION AND TWO MODIFIED VERSIONS OF BOUNDARY CONDITION

This boundary equation will give the form of the Euclidean region in the plane of minisuperspace variables (a, σ). We can approximate the regiones of the potential barrier $U(a, \sigma)$ into three parts according to the range of the scale factor a: call Region A for term $-\frac{Q^2}{2a^2}$ dominates as $a \to 0$. Region C for the term $-a^4 V(\sigma)$ dominates as $a \to \infty$. The intermediate Region B for the term $6a^2$ dominates. Here we can see that for Region A, $U(a, \sigma) < 0$, it is thus a Lorentzian region. Region B with $U(a, \sigma) > 0$ is a Euclidean region, and Region C with $U(a, \sigma) < 0$ is a Lorentzian one again. Notice that the presence of the charge Q modifies the Euclidean region different with the usual case, that is, no Lorentzian region as $a \to 0$ before the Euclidean region. Hence and therefore, the superpotential $U(a, \sigma)$ displays a new and interesting feature in the minisuperspace.

In contrast with the picture of "tunneling from nothing" and with the "no-boundary condition" for the wave function of the Universe, we have ones which can go into the Euclidean region from one side and go out from the other. These features require some redefinition (Kamenshcik *et al.*, 1995) of the original Vilenkin's "tunneling" boundary conditions (Vilenkin, 1984, 1988a,b) and Hartle–Hawking's no-boundary condition (Hartle, 1991; Hartle and Hawking, 1983). First, in the tunneling boundary condition approach, there will be no picture corresponding to "tunneling from nothing." The wave function of the Universe will evolve along the scale factor direction from an initial Lorentzian region to a final Lorentzian region, bypassing a Euclidean region. Thus the authors extends the Vilenkin's boundary condition to the prescription of taking only the outgoing mode (expanding) of the wave function of the Universe in the Lorentzian region. We will call it the Vilenkin-like boundary condition.

It can be observed in the superpotential (1.22) that the presence of the charge term Q prevents the regularity of matter fields at $a = 0$. Again we need an extension of the original Hartle–Hawking proposal, that is, the Hartle–Hawking-like boundary condition that the wave function of the Universe should be an exponentially growing function of the scale factor a in the classically forbidden region. In the following we will make use of these two modified versions of boundary condition to apply in our case of charged generalized Brans–Dicke gravity.

3. WAVE FUNCTIONS OF THE MODEL

The lapse function N is not of physical relevance classically since we can rescale the proper time parameter $d\tau = N\,dt$ or choose it to be $N = 1$. Write the Lagrangian depending only on two minisuperspace variables a and σ, and their derivatives,

$$L = (-6\dot{a}^2 a + 6a) + \frac{1}{2}a^3\dot{\sigma}^2 - \frac{Q^2}{2a^3} - a^3 V(\sigma) \tag{3.1}$$

By using the Lagrangian (3.1) and the Dirac's quantizing procedure, the operator version of the Hamiltonian constraint (1.21) becomes

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial a^2} - \frac{1}{a^2}\frac{\partial^2}{\partial \sigma^2} - \left(6a^2 - \frac{Q^2}{2a^2} - a^4 V(\sigma)\right)\right)\Psi = 0 \tag{3.2}$$

If $Q = 0$, the case for the real scalar field will be restored. We are going to look into the case where $Q \neq 0$. As the scale factor a goes from zero to infinity, some solution to the Wheeler–DeWitt equation will cross the initial classical allowed Lorentzian region with $U < 0$, and then the classically forbidden Euclidean region with $U > 0$, and finally the classically allowed region again.

We can study the qualitative property by assuming the "slow-rolling approximation" (Kolb and Turner, 1990) of inflationary cosmology. In this approximation, the second derivatives of the wave function Ψ with respect to the dilaton field σ, $\frac{\partial^2\Psi}{\partial\sigma^2} \approx 0$. Then the Wheeler–DeWitt Eq. (3.2) will be of the form

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial a^2} - U(a,\sigma)\right) = 0 \tag{3.3}$$

The general solution to Eq. (3.3) in the semiclassical WKB approximation in the range (a_0, a) is a linear combination of the terms.

$$\Psi \propto \exp\left(\pm i \int_{a_0}^{a} (-U)^{1/2} da'\right) \tag{3.4}$$

The positive and negative signs characterize the ingoing and outgoing waves respectively. We will illustrate in the following: $\pi_a\Psi > 0$ (outgoing) for negative sign while $\pi_a\Psi < 0$ (ingoing) for positive sign. Actually, the outgoing wave is defined as expanding Universe and ingoing wave as contracting Universe. Although we cannot evaluate the integral in (3.4) exactly, it is possible to approximate the analytic solutions in the respective regions we have just defined before. Notice that the boundary between Region A and Region B is at $a_1^2 \approx \frac{Q}{\sqrt{12}}$, whereas the boundary between Region B and Region C is at $a_2^2 \approx \frac{6}{V(\sigma)}$. Let us consider separately the three regions.

Region A: The superpotential is approximated to $U(a, \sigma) \approx -\frac{Q}{2a^2}$ and (3.4) gives in the range (a, a_1)

$$\Psi_{\rm A} \propto \exp\left(\pm i \frac{Q}{\sqrt{2}} \ln\left(\frac{a}{a_1}\right)\right) \tag{3.5}$$

This is an oscillating wave function as expected in Lorentzian region. Region B and Region C can be solved together. $U(a, \sigma) \approx 6a^2 - V(\sigma)a^4$

$$\Psi_{\rm BC} \propto \exp\left(\pm \frac{(6 - V(\sigma)a^2)^{3/2} - \left(6 - V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) \tag{3.6}$$

When $a^2 < a_2^2 \approx \frac{6}{V(\sigma)}$, which is under the barrier, the wave function is of exponential form whereas an oscillating one for $a^2 > a_2^2 \approx \frac{6}{V(\sigma)}$ which is in classical region.

Imposing the Vilenkin-like boundary condition on the wave function, only classically expanding solution is to be taken at the Euclidean–Lorentzian boundary. It selects the outgoing wave in the Lorentzian region. On the other hand, the Hartle–Hawking-like boundary condition will result in the exponentially increasing solution under the barrier. It selects the outgoing wave in the classically forbidden region. To be precise in the following, by using the WKB connection formula, we will write down the wave functions in different regions explicitly.

Imposing the Vilenkin-like boundary condition implies that only an outgoing wave should be present in the classically allowed region:

Region A:

$$\Psi_{\rm A} \propto \exp\left(-i \frac{Q}{\sqrt{2}} \ln\left(\frac{a}{a_1}\right)\right) \tag{3.7}$$

Region B:

$$\Psi_{\rm B} \propto \exp\left(\frac{(6 - V(\sigma)a^2)^{3/2} - \left(6 - V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) - \frac{i}{2}\exp\left(-\frac{(6 - V(\sigma)a^2)^{3/2} - \left(6 - V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) \tag{3.8}$$

The first term is of decreasing exponential while the second term is of growing exponential where they have comparable amplitudes at the nucleation point $a = a_2$, but away from that point the decreasing exponential dominates.

Region C:

$$\Psi_C \propto \exp\left(-\frac{i(V(\sigma)a^2-6)^{3/2}-\left(6-V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) \tag{3.9}$$

As we can see, WKB solution is proportional to the form e^{-iS} where $S = (V(\sigma)a^2 - 6)^{3/2}$ it selects the outgoing mode at the boundary of the Euclidean region. Imposing the Hartle–Hawking-like boundary condition which is specified by requiring that $\exp(-S_E)$ in the Euclidean underbarrier region:

Region A:

$$\Psi_A \propto \exp\left(i\frac{Q}{\sqrt{2}}\ln\left(\frac{a}{a_1}\right)\right) - \exp\left(-i\frac{Q}{\sqrt{2}}\ln\left(\frac{a}{a_1}\right)\right) \tag{3.10}$$

Region B:

$$\Psi_B \propto \exp\left(-\frac{(6-V(\sigma)a^2)^{3/2}-\left(6-V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) \tag{3.11}$$

Region C:

$$\Psi_C \propto \exp\left(\frac{i(V(\sigma)a^2-6)^{3/2}-\left(6-V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) - \exp\left(-\frac{i(V(\sigma)a^2-6)^{3/2}-\left(6-V(\sigma)a_1^2\right)^{3/2}}{3V(\sigma)}\right) \tag{3.12}$$

One can notice that the wave function in the complex scalar field case is not much different with the real scalar field case except $a_1 \neq 0$. Having obtained the WKB wave functions in different regions for these two boundary conditions, we are now at a position to compare their corresponding probabilities of nucleation by applying a sensible probability measure.

4. PREDICTIONS OF CGBD MODEL

The final task we have to do on this model is to extract prediction from it. Let us now compare the no-boundary and tunneling wave functions. Firstly, it is observed that both wave functions are peaked about the same set of solutions to the field equations, namely those satisfying the Hamilton-Jacobi equation.

$$\frac{\partial S}{\partial t} + U(a, \sigma) = 0 \tag{4.1}$$

These solutions are initially inflationary with $a \propto \exp(\sqrt{V(\sigma)t})$. Although all those solutions undergo some inflation, the amount by which they inflate depends on $V(\sigma)$. The probability will give the distribution of the initial values of the nucleated Universe and the tunneling probability can be estimated as for Vilenkin-like boundary condition,

$$dP_{\mathrm{T}} \propto \exp\left(-\frac{2\left(6 - V(\sigma)\frac{Q}{\sqrt{12}}\right)^{3/2}}{3V(\sigma)}\right) \tag{4.2}$$

and for Hartle–Hawking-like boundary condition,

$$dP_{\mathrm{HH}} \propto \exp\left(+\frac{2\left(6 - V(\sigma)\frac{Q}{\sqrt{12}}\right)^{3/2}}{3V(\sigma)}\right) \tag{4.3}$$

These can be interpreted as the probability distributions for the initial values of $V(\sigma)$ in the nucleated Universe. The probabilities for these two approaches also differ by a crucial sign in front of it. We can see that if there is no charge $Q = 0$, the result will resemble to the case of real scalar field. Once it nucleates, the Universe immediately begins a de Sitter inflationary expansion. In slow-rolling approximation $\dot{\sigma} \approx 0$, $V(\sigma) \to \Lambda$ where the vacuum energy density dominates. In order for sufficient inflation to solve the problems arise in hot big bang theory, the vacuum energy density of the scalar field should be large enough to drive the bubble into an exponential expansion. Below we will illustrate the probability distributions for two redefined versions of boundary conditions respectively in Figs. 1 and 2. Alongside, Figs. 3 and 4 for the probability distributions in the

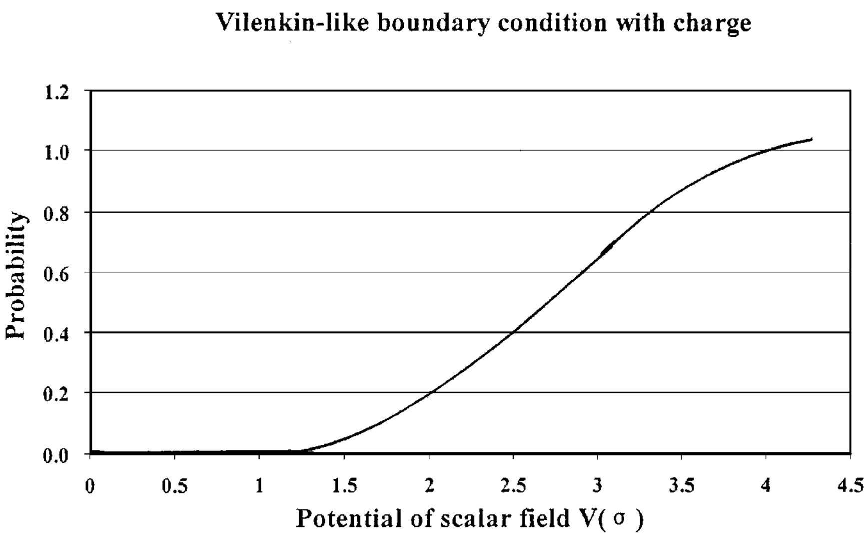

Fig. 1. The probability distribution of the wave function in Vilenkin-like boundary condition. It gives a large probability for large value of potential of dilaton field $V(\sigma)$.

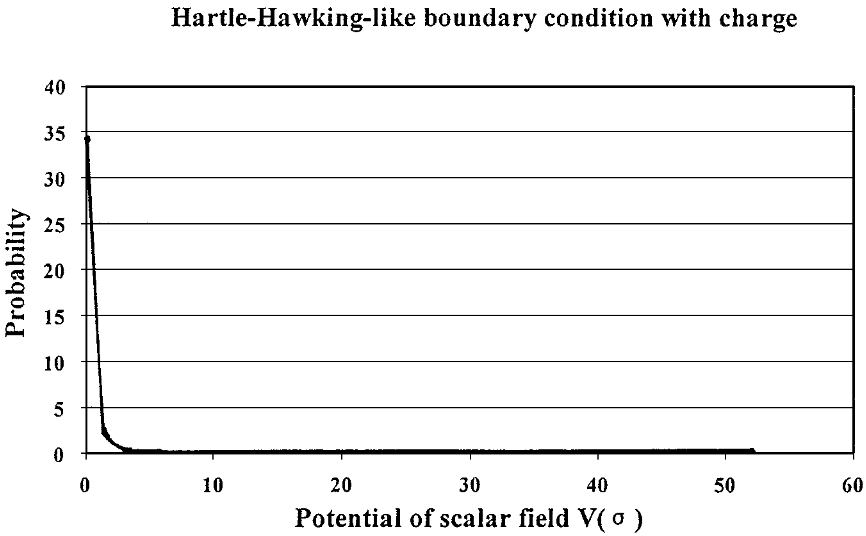

Fig. 2. The probability distribution of the wave function in Hartle–Hawking-like boundary condition. It gives a low probability for large value of potential of dilaton field $V(\sigma)$.

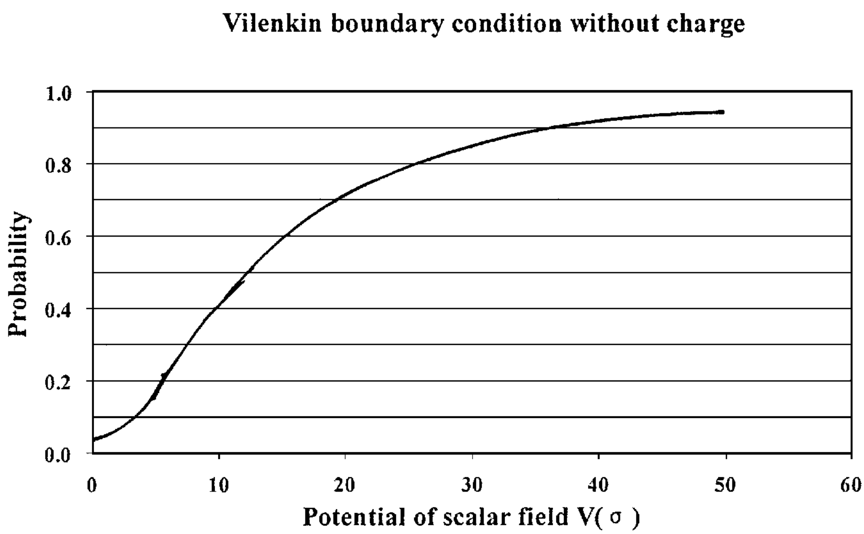

Fig. 3. The probability distribution of the wave function in the original Vilenkin boundary condition. It gives a large probability for large value of potential of dilaton field $V(\sigma)$.

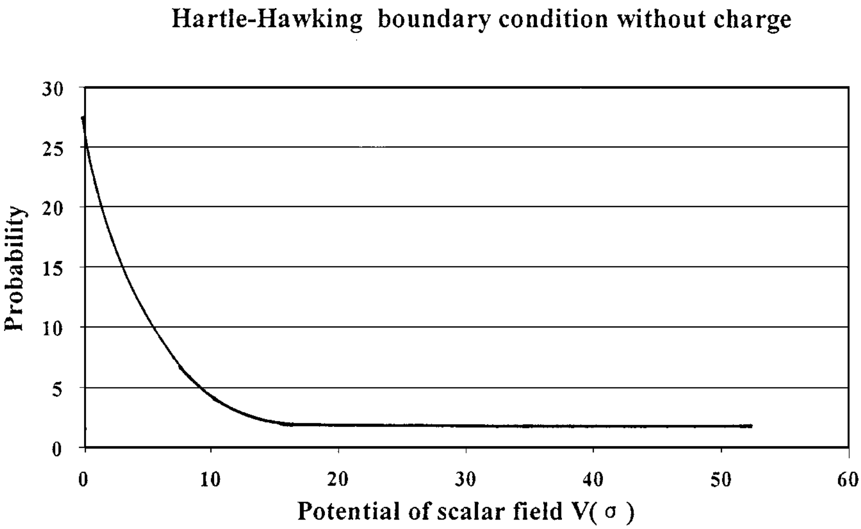

Fig. 4. The probability distribution of the wave function in the original Hartle–Hawking boundary condition. It gives a low probability for large value of potential of dilaton field $V(\sigma)$.

usual real scalar field are shown in parallel. We notice that in our case of charged generalized Brans–Dicke gravity, the probability distributions are very similar to the case of real scalar field. This is mainly due to the fact that the term containing the charge in the potential is independent of the dilaton scalar field in our case. Thus, even in the presence of the charge term the probability distributions are not affected very much compared with the original real scalar field.

As explained before, the Vilenkin-like boundary condition predicts that the Universe is most likely to nucleate with the largest possible vacuum potential energy and thus results in a larger amount of inflation. This is a welcome news of initial condition for inflation. In other words, in charged generalized Brans–Dicke gravity in which a quasi-fundamental constant charge Q is introduced, inflation is favored by the Vilenkin-like boundary condition. On the other hand, the Hartle–Hawking-like boundary condition, because of the opposite sign in the probability distribution, a crucial difference, predicts initial conditions nucleating with the smallest possible vacuum energy density, which disfavors inflation.

5. CONCLUSION AND OUTLOOK

The second law of thermodynamics tells us that the Universe evolves from more ordered to more disordered states. It suggests that the Universe has started in a very nonrandom configuration of high order and symmetry. But, what is the physical principle that determines the initial state of the Universe? Theory of inflation provides some solutions to the classic puzzles in the standard cosmology. Nevertheless, the most crucial requirement for such sufficient inflation is a large vacuum energy density presented at the beginning of the inflationary era. This is assumed rather than predicted. In this thesis, we have reviewed the attempts to understand the initial conditions of the Universe in the framework of quantum cosmology, namely, Vilenkin's tunneling proposal and Hartle–Hawking no-boundary proposal. However, because of the infinite dimension of superspace, neither of these two approaches have yet been translated beyond minisuperspace.

The Brans–Dicke theory is used throughout the thesis as an application of quantum cosmology. We have used the Pauli metric as the physical metric and obtain the Lagrangian in Pauli frame. Also, a $U(1)$ charge group is coupled to the scalar field. It predicts the right initial conditions of the inflation Universe in Vilenkin's boundary proposal. In quantum cosmology with nonlinear Born–Infeld scalar field, Vilenkin's tunneling approach predicts that the Universe nucleates with the largest possible vacuum energy and interactions of particles of the nonlinear scalar field are the largest possible, giving out the right initial condition for inflation (Harko and Cheng, 1999). it seems at this moment that both proposals are still under criticism. Different ones are applicable to different cases. No one can fully discriminate between these two ideas. We should be aware that the two wave functions are far from being rigorously defined mathematical objects. Except for the simplest models, the actual calculations of these wave functions involve additional assumptions which may appear reasonable, but are not really well justified (Vilenkin, 1998). Actually, quantum cosmology can only give a probability distribution for the initial states of the Universe and, on the other hand, we have only one Universe. We could guess the best that we are now living in a "typical" Universe which has started somewhere near the maximum of the probability distribution. This is really an issue of interpretation of wave function.

In the prescription of no-boundary condition, the Euclidean action $S_{\mathrm{E}}(g_{\mu\nu}, \phi)$ for gravitation coupled to matter field is unbounded below in general. Hence the Feymann sum over the compact manifolds will be divergent. Hartle and Hawking (1983) then proposed that the sum should be taken over a class of complex geometries, not pure Lorentzian, nor pure Euclidean. A complex contour is essential in no-boundary condition. However, many different complex convergent contours are possibly available, and correspondingly there are many different no-boundary wave functions. These will result in predictions of many different Universes. However, we still lack a principle for fixing this wave function of the Universe.

We state the last outlook in quantum cosmology as a final remark. Extracting the predictions of initial conditions and comparing them with observations is a central problem in quantum cosmology. However, the probabilities for those large scale features of the Universe have been explored in highly simplified models only and in limited regions of the configuration space. Among them, the spectrum of initial quantum fluctuations is a quite successful achievement of quantum cosmology. But much has to be done to extend the theory to the whole of configuration space with greater accuracy, generality, and more precise quantum mechanical interpretation. These are some problems for the twenty-first century from Hartle's paper, *Quantum Cosmology: Problems for the 21st century at 1998*.

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